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Event-by-event fluctuations in heavy-ion collisions and the quark–gluon string model

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Abstract. We apply dynamical string models of heavy-ion collisions at high energies to the analysis of event-by-event fluctuations. Attention is devoted mainly to a new variable proposed for studying "equilibration" in heavy-ion collisions. Recent results of the NA49 collaboration at CERN SPS are compared with predictions of the quark–gluon string model (QGSM), which gives a good description of different aspects of multiparticle production for collisions of nucleons and nuclei. It is shown that the new observable and other results of the NA49 analysis of event-by-event fluctuations are correctly reproduced in the model. We discuss dynamical effects responsible for these fluctuations and give the predictions for p–p, p–Pb and Pb–Pb collisions at RHIC and higher energies.

1 Introduction

An event-by-event analysis of heavy-ion collisions can give important information on the dynamics of these processes. In [1] it was proposed that an event-by-event analysis of transverse momentum fluctuations be used as a method for the study of "equilibration" in high-energy nucleus–nucleus collisions. For this purpose, a special variable Φ has been introduced in [1]. Let us recall briefly the definition of Φ as well as the method for studying event-by-event fluctuations of transverse momenta of produced particles introduced in [1]. It was proposed that each particle be defined in a given event a variable $z_i = p_{\mathrm{T}i} - \langle p_{\mathrm{T}} \rangle$, where $p_{\mathrm{T}i}$ is the transverse momentum of the particle i and $\langle p_{\mathrm{T}} \rangle$ is the mean transverse momentum of particles averaged over all events. Through the use of z_i, the quantity $Z = \Sigma_{i=1}^{N} z_i$ is defined, where N is the total number of particles in the event. If nucleus–nucleus collisions can be considered as a superposition of independent nucleon–nucleon collisions, then it can be shown [1] that

$$\frac{\langle Z^2 \rangle_{\mathrm{AA}}}{\langle N \rangle_{\mathrm{AA}}} = \frac{\langle Z^2 \rangle_{\mathrm{NN}}}{\langle N \rangle_{\mathrm{NN}}} \quad . \tag{1}$$

A derivation of this result is given in Appendix A.

The averaging in (1) is over all events in a given kinematical region. It was further proposed in [1] that the degree of fluctuations be characterized by the variable

$$\Phi = \sqrt{\frac{\langle Z^2 \rangle}{\langle N \rangle}} - \sqrt{\langle z^2 \rangle}, \tag{2}$$

where $\langle z^2 \rangle$ is the second moment of the single particle inclusive z distribution. The quantity $\langle z^2 \rangle$ corresponds to purely statistical fluctuations and is determined by mixing particles from different events. It was emphasized in [1] that if nucleus–nucleus collisions were a simple superposition of independent nucleon–nucleon collisions, then the variable Φ would be the same as in the nucleon–nucleon case (see Appendix A). In nucleon–nucleon collisions, the quantity Φ is different from zero because of dynamical correlations and, in particular, the dependence of $\langle p_{\mathrm{T}} \rangle$ on the number of produced particles. It was proposed that the possible decrease of the quantity Φ in A–A collisions be attributed to the effects of equilibration. The problem of equilibration in high-energy heavy-ion collisions is very important if one is to understand the conditions for quark–gluon plasma formation. Recent experimental results on event-by-event analysis of Pb–Pb collisions at CERN SPS by the NA49 collaboration [2] show that the value of Φ is substantially smaller than expected in the case of independent nucleon–nucleon collisions, and its smallness has been considered as an indication of equilibration in the system.

This result has been discussed in the framework of different theoretical models. It has been shown [3] that the increase in transverse momenta of hadrons due to multiple rescatterings leads to a substantial increase of Φ. Incorporating this effect in the model of [4], one finds an even stronger disagreement with the NA49 result. Additionally, it has been demonstrated that string fusion leads also to an increase of Φ in disagreement with experiment [5,6]. The results on the influence of final-state interactions on the observable Φ are contradictory: In [5], it was shown that final-state interactions in the framework of the string model of [5] have a small effect and do not allow agreement with experiment to be reached, while in [7] it has been argued that final-state interactions in the framework of the UrQMD model are essential and can decrease Φ to a value consistent with experiment. On the other hand, it has been shown in [8] that, in the case of fully equilibrated hadronic gas made mostly of pions, one expects large positive values for the variable Φ not consistent with the experimental observation.

In this paper, we study event-by-event fluctuations using the Monte Carlo formulation [9] of the quark–gluon string model (QGSM) [10]. The QGSM and the Dual Parton Model (DPM) [11]) are closely related dynamical models based on 1/N expansion in QCD, string fragmentation, and Reggeon calculus. They give a good description of many characteristics of multiparticle production in hadron–hadron, hadron–nucleus and nucleus–nucleus collisions (for a review, see [12]). Nuclear interactions in this model are treated in the Glauber–Gribov approach. It will be shown that the model reproduces the results of the event-by-event analysis of the NA49 experiment for the quantity Φ as well as for other fluctuations observed in this experiment [2]. We analyze the reason for the decrease of the quantity Φ from p–p to Pb–Pb collisions seen by the NA49 experiment and come to the conclusion that the quantity Φ is sensitive to many details of the interaction and can hardly be considered as a good measure of equilibration in the system. We predict a strong increase of the value of Φ at energies of RHIC and higher. The model also gives definite predictions for event-by-event fluctuations in p–Pb collisions.

Table 1. The results of the Monte Carlo calculation for the quantities $\sqrt{\langle Z^2\rangle/\langle N\rangle}$ (MeV), $\sqrt{\langle z^2\rangle}$ (MeV), and Φ (MeV) for p–p, p–Pb, and central Pb–Pb-collisions at SPS ($\sqrt{s}$ = 19.4 GeV), RHIC ($\sqrt{s}$ = 200 GeV), and higher ($\sqrt{s}$ = 540 GeV and 1 TeV) energies. The results at 1 TeV have only an indicative value (see main text)

	$\sqrt{s}$ (GeV)	$\sqrt{\frac{\langle Z^2\rangle}{\langle N\rangle}}$ (MeV)	$\sqrt{\langle z^2\rangle}$ (MeV)	Φ (MeV)
p–p	19.4	244.5	235.5	9.0
p–Pb	19.4	243.5	243.0	0.5
Pb–Pb	19.4	265.6	263.2	2.4
p–p	200	387.0	310.6	76.4
p–Pb	200	433.7	367.8	65.9
Pb–Pb	200	508.9	429.4	79.5
p–p	540	450.1	323.6	126.5
p–Pb	540	524.3	397.7	126.6
Pb–Pb	540	622.6	475.2	147.4
p–p	1000	455.5	324.4	131
p–Pb	1000	524.5	397.5	127
Pb–Pb	1000	704.2	484.3	220

2 Analysis of event-by-event fluctuations of transverse momenta

The model of independent N–N collisions for nucleus–nucleus interactions used in [1] is an extremely oversimplified one. The Glauber model at high energies is not equivalent to independent N–N collisions even for N–A interactions. The space-time picture of hadron–nucleus interactions at high energies is absolutely different from a simple picture of successive reinteractions of an initial hadron with nucleons of the nucleus (see, e.g., [13–15]). For nucleus–nucleus interactions there are extra correlations [14,16]. The model of independent N–N collisions does not even satisfy energy-momentum conservation, because a nucleon of one nucleus can not interact inelastically several times with nucleons of another nucleus having the same energy at each interaction.

In the QGSM, as well as the DPM, the effects of multiple interactions in hadron–nucleus and nucleus –nucleus collisions are taken into account in an approach based on the topological expansion in QCD [12]. Probabilities of rescatterings are calculated in the framework of the Glauber–Gribov theory, and multiparticle configurations in the final state are determined using AGK [17] cutting rules. In these models, the Pomeron is related to the cylinder-type diagrams, which correspond to the production of two chains of particles due to decays of two qq–q strings. Multi-Pomeron exchanges are related to multicylinder diagrams, which produce extra chains of type q–$\overline{\text{q}}$. They are especially important in interactions with nuclei. Fragmentation of strings into hadrons is described according to "Regge-counting rules" [18], which give correct triple-Regge and double-Regge limits of inclusive cross sections. All conservation laws (including energy-momentum conservations) are satisfied in this approach.

Let us emphasize that at energies $\sqrt{s} \sim 10$ GeV, the cylinder-type diagrams give the dominant contributions for N–N collisions. Extra q–$\overline{\text{q}}$ chains due to multicylinder diagrams have rather small length in rapidity (short chains) and do not lead to substantial contributions to particle production. In nucleon–nucleus and nucleus–nucleus collisions, the number of short chains is strongly increased as compared to the nucleon–nucleon case (the number is proportional to a number of collisions) and this should be taken into account in any realistic calculations of multiparticle production on nuclei. This means that for p–A and A–B collisions, there are extra "clusters" of particles (short chains, of type q–$\overline{\text{q}}$) as compared to the nucleon–nucleon interactions clusters (long chains connecting valence quarks and diquarks of the colliding nucleons).

So we come to the conclusion that in the relativistic Glauber–Gribov dynamics, the characteristics of final particles in N–A and A–B collisions can not be expressed in terms of N–N collisions only, as it was assumed in [1]; thus (1) is not valid in general.

The results of the Monte Carlo calculation for the quantity Φ are shown in Table 1 for p–p, p–Pb, and central Pb–Pb collisions at SPS energies ($\sqrt{s}$ = 19.4 GeV) and at RHIC ($\sqrt{s}$ = 200 GeV). Predictions of the model for Φ are quite different for these two energies. At SPS, there is a strong reduction of the quantity Φ for nuclear collisions as compared to N–N collisions, while at RHIC, the quantity Φ is predicted to be much larger than at SPS and about the same for Pb–Pb and p–p collisions. At LHC energies, the value of Φ obtained in our model is 160 MeV for p–p and even larger for Pb–Pb collisions (the Monte Carlo code we use does not allow one to calculate precise values of the correlations at LHC energies, because of a too-large number of particles produced in each event, so in Table 1 we give predictions of the model at $\sqrt{s} = 540$ GeV and $\sqrt{s} = 1$ TeV to show the energy dependence of fluctuations). The results for SPS energies are in a reasonable agreement with experimental data of the NA49 Collaboration [2]. The result for p–p interactions at this energy is even higher than the estimate, based on the dependence of $\langle p_{\mathrm{T}} \rangle$ on the number of charged particles, given in [1]. The model reproduces this correlation reasonably well and shows that this is not the only source of fluctuations leading to a nonzero value of Φ. We find that the quantity Φ is sensitive also to other types of correlations and, in particular, to the correlations related to conservation of p_{T} in the process.

Let us note that the quantity Φ at SPS energies is very small and is defined in (2) as the difference of two large numbers (see Table 1), so it is very sensitive to all details of dynamical models. Because of its smallness, it is difficult for a good accuracy to be obtained in a Monte Carlo calculation of this quantity (especially for nucleus–nucleus collisions, where the maximum statistics possible in the Monte Carlo is ≈ 5000 events). In order to increase the statistics and to reduce this uncertainty, we give in Table 1 results obtained for the total rapidity interval, and the experimental data of the NA49 Collaboration, which were obtained in a fixed rapidity interval $4 < y_{\pi}^{lab.} < 5.5$. The error in the values of Φ in Table 1 is about 1 MeV for the lowest energies, and for increases at high energy. Other properties of event-by-event fluctuations observed by the NA49 Collaboration [2] were calculated under the conditions of the experiment and are reproduced by the model reasonably well, as is shown in Fig. 1.

It follows from Table 1 that at SPS energies, there is an increase of the quantity $\sqrt{\langle Z^2 \rangle / \langle N \rangle}$ from p–p to Pb–Pb collisions, but there is an even larger increase for $\sqrt{\langle z^2 \rangle}$ due to the increase of $\langle p_{\mathrm{T}} \rangle$ and to a change in the form of the p_{T} distribution. The effect of the correlations between $\langle p_{\mathrm{T}} \rangle$ and the number of charged particles due to rescatterings is, to a large extent, compensated at these energies by energy-momentum conservation effects. As a result, we find no dependence of $\langle p_{\mathrm{T}} \rangle$ on n_{ch} for Pb–Pb collisions at SPS. For RHIC energies, a strong increase in the values of both $\sqrt{\langle Z^2 \rangle / \langle N \rangle}$ and especially of Φ is predicted (see Table 1). At these energies, the increase of average transverse momentum with the number of rescatterings becomes very important in p–p interactions and is reproduced by the QGSM (Fig. 2a). It is shown in Fig. 2b that an increase of $\langle p_{\mathrm{T}} \rangle$ with multiplicity is predicted at these energies even for Pb–Pb collisions, although the effect is less pronounced for heavy-ion collisions than for p–p. At LHC energies, all these effects will be stronger than at RHIC and will produce an increase of the quantity Φ in p–p, p–A and A–A collisions as energy increases. The predictions of the model can be easily tested in future experiments at RHIC and LHC.

Finally, in Appendix A we give as an illustrative example the results in a model with two types of clusters. This model is a generalization of the single-cluster model of [1], and is much closer to QGSM and DPM. In this sense, this two-cluster model is useful for the understanding of the physical origin of our main results.

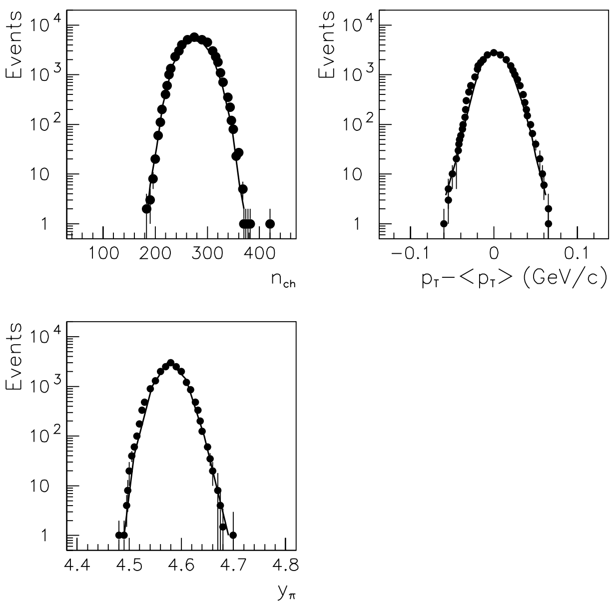

Fig. 1. Event spectra characterizing the multiplicity, transverse momentum and rapidity distribution of charged particles per event for Pb–Pb collisions at $P_{\mathrm{lab}} = 158$ A GeV/c and $b \leq 3.5$ fm in the rapidity interval $4 \leq y \leq 5.5$. The full lines are the Monte Carlo results. Experimental data are from [2]

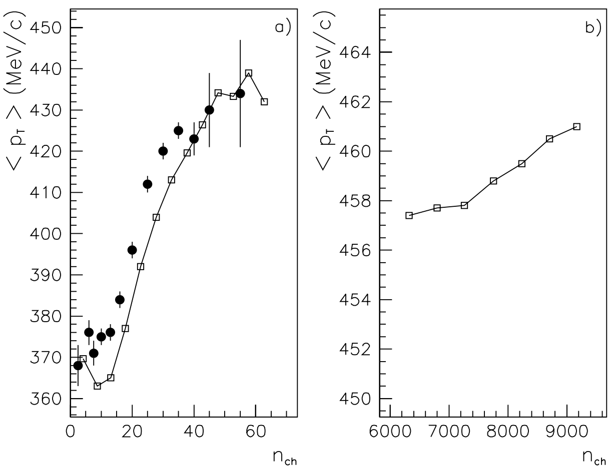

Fig. 2a,b. The dependence of the average transverse momentum on the multiplicity of charged particles in the window $|\eta| \leq 2.5$ at $\sqrt{s} = 200$ GeV for $\mathrm{p} - -\overline{\mathrm{p}}$ collisions compared to experimental data [19] **a** and for Pb–Pb central collisions **b**

3 Conclusions

We have shown, in a Monte Carlo version of the QGSM, that at SPS energies, the quantity Φ, characterizing event-by-event transverse momentum fluctuations, decreases from $\Phi \sim 9$ MeV in p–p collisions to $\Phi \sim 2$ MeV in central Pb–Pb collisions. This result for Pb–Pb collisions agrees with the measurement of the NA49 Collaboration. In [1], such a decrease between p–p and Pb–Pb was considered to be a test of equilibration of the dense system produced in central heavy-ion collisions. We have obtained the same result in the framework of an independent string model.

At RHIC energies, we predict an increase in the value of Φ ($\Phi = 75 \div 80$ MeV). In this case Φ will be approximately the same in p–p and central Pb–Pb collisions. At higher energies, the value of Φ is predicted to be larger and to increase from p–p to central Pb–Pb collisions.

Our analysis indicates that the quantity Φ can hardly be considered as a good measure of equilibration in the system. However, it can be used as a sensitive test of dynamical models.

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Appendix A

Here we will consider a simplified model of multiparticle production with two types of clusters; this is a generalization of the model of [1], where clusters of only a single type were produced. As discussed above, these clusters correspond to qq–q chains (clusters of the first type) and q–$\overline{\text{q}}$ chains (clusters of the second type). Nucleon–nucleon collision at SPS energies can be described with a good accuracy by the production of two clusters of the first type, while for proton–nucleus and nucleus–nucleus interactions, production of the second type of clusters is important even in this energy range. For independent production of k_1 clusters of the first type and k_2 clusters of the second type, with average transverse momenta $\langle p_\mathrm{T}\rangle_i$ and multiplicity $\langle n\rangle_i$ for the cluster i, the following results for $\langle Z\rangle$ and $\langle Z^2\rangle$ can be obtained:

$$\langle Z\rangle_{k_1k_2} = k_1\langle Z\rangle_1 + k_2\langle Z\rangle_2 \tag{A.1}$$

where

$$\langle Z\rangle_i = \langle n\rangle_i(\langle p_\mathrm{T}\rangle_i - \langle P_\mathrm{T}\rangle)$$

and

$$\langle P_\mathrm{T}\rangle = \frac{(\langle k_1\rangle\langle n_1\rangle\langle p_\mathrm{T}\rangle_1 + \langle k_2\rangle\langle n\rangle_2\langle p_\mathrm{T}\rangle_2)}{(\langle k_1\rangle\langle n\rangle_1 + \langle k_2\rangle\langle n\rangle_2)}.$$

$$\begin{aligned}\langle Z^2\rangle_{k_1k_2} &= k_1\langle Z^2\rangle_1 + k_2\langle Z^2\rangle_2 + k_1(k_1-1)\langle Z\rangle_1^2 \\ &\quad + k_2(k_2-1)\langle Z\rangle_2^2 + 2k_1k_2\langle Z\rangle_1\langle Z\rangle_2\end{aligned} \tag{A.2}$$

The expression for $\langle Z^2\rangle_{k_1k_2}$ can be rewritten as

$$\begin{aligned}\langle Z^2\rangle_{k_1k_2} &= k_1(\langle Z^2\rangle_1 - \langle Z\rangle_1^2) \\ &\quad + k_2(\langle Z^2\rangle_2 - \langle Z\rangle_2^2) + \langle Z\rangle_{k_1k_2}^2 \ .\end{aligned} \tag{A.3}$$

For the quantity Φ in (2) it is important that contrary to the case of a single cluster, the expression for $\langle Z^2\rangle$ contains negative terms proportional to $\langle Z\rangle_i^2$.

Next we take the average over the number of produced clusters with some distribution $P_{k_1k_2}$

$$\begin{aligned}\langle\langle Z^2\rangle_{k_1k_2}\rangle &= \sum_{k_1,k_2} P_{k_1k_2}\langle Z^2\rangle_{k_1k_2} \\ &= \langle k_1\rangle(\langle Z^2\rangle_1 - \langle Z\rangle_1^2) + \langle k_2\rangle(\langle Z^2\rangle_2 - \langle Z\rangle_2^2) \\ &\quad + \langle\langle Z\rangle_{k_1k_2}^2\rangle \ .\end{aligned} \tag{A.4}$$

In the following, we will denote this averaging simply by $\langle Z^2\rangle$. We shall concentrate on p–A collisions. In this case, it is easy to show that the last term in (A.4) is small and can be neglected. To prove this, we note that for p–A collisions, $k_2 = k_1 - 2$ [12] with $k_1 = \overline{\nu} + 1$, where $\overline{\nu}$ is the average number of collisions, thus,

$$\begin{aligned}&\langle\langle Z\rangle_{k_1k_2}^2\rangle - \langle\langle Z\rangle_{k_1k_2}\rangle^2 \\ &\quad = (\langle k_1^2\rangle - \langle k_1\rangle^2)(\langle Z\rangle_1 + \langle Z\rangle_2)^2 \ .\end{aligned} \tag{A.5}$$

Taking into account that (for a fixed impact parameter) the distribution in k_1 is of a Poisson type with $(\langle k_1^2\rangle - \langle k_1\rangle^2) = c_1\langle k_1\rangle$, and that $\langle\langle Z\rangle_{k_1k_2}\rangle = \langle k_1\rangle\langle Z\rangle_1 + \langle k_2\rangle\langle Z\rangle_2 = \langle k_1\rangle\langle Z\rangle_1 + (\langle k_1\rangle - 2)\langle Z\rangle_2 = 0$, we obtain:

$$\langle\langle Z\rangle_{k_1k_2}^2\rangle = \frac{4c_1\langle Z\rangle_2^2}{\langle k_1\rangle} \ . \tag{A.6}$$

For large values of $\langle k_1\rangle$, this quantity is much smaller than the other terms in the right-hand side of (A.4).

The expressions for the quantities $\langle N\rangle$ and $\langle z^2\rangle$ that enter into the definition of Φ are self-evident:

$$\langle N\rangle = \langle k_1\rangle\langle n_1\rangle + \langle k_2\rangle\langle n_2\rangle \tag{A.7}$$

and

$$\langle z^2\rangle = \frac{\langle k_1\rangle\langle n\rangle_1\langle z^2\rangle_1 + \langle k_2\rangle\langle n\rangle_2\langle z^2\rangle_2}{\langle k_1\rangle\langle n\rangle_1 + \langle k_2\rangle\langle n\rangle_2} \ . \tag{A.8}$$

Let us denote $\frac{\langle Z^2\rangle_i}{\langle n\rangle_i}$ as $\langle z^2\rangle_i(1+\delta_i)$ and $\frac{\langle Z\rangle_i^2}{\langle Z^2\rangle_i} \equiv \gamma_i$, $\frac{\langle k_2\rangle\langle n\rangle_2}{\langle k_1\rangle\langle n\rangle_1} \equiv \alpha$. Taking into account that $\delta_i \approx 2\frac{\Phi_i}{\sqrt{z_i^2}}$ and γ_i are much smaller than unity, we obtain the following approximate expression for Φ:

$$\Phi = \frac{[(\delta_1 - \gamma_1)\langle z^2\rangle_1 + (\delta_2 - \gamma_2)\alpha\langle z^2\rangle_2]}{2\sqrt{A}} \tag{A.9}$$

where $A = (1+\alpha)(\langle z^2\rangle_1 + \alpha\langle z^2\rangle_2)$. It is important that the terms proportional to γ_i give negative contributions to Φ and can substantially decrease the value of Φ.

For the case of clusters of the same type ($\gamma_i = 0, \langle Z^2 \rangle_1 = \langle Z^2 \rangle_2, \delta_1 = \delta_2$) we obtain:

$$\Phi = \frac{\delta_1}{2} \sqrt{\langle z^2 \rangle} \tag{A.10}$$

for both for p–p and p–A. We recover in this way the result of [1]. The discussion in this appendix has been restricted to p–A interactions. The situation is more complicated in A–B collisions. Actually, even for p–A, we do not claim that the effect discussed in this appendix is the main reason for the decrease of Φ obtained in the Monte Carlo calculations (see Table 1) between p–p and p–A collisions at SPS energies. Nevertheless, our example illustrates the important effect that a modification of the model (i.e., going from one to two types of clusters) can have on the quantity Φ.

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